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On Validation of a Popular Sport Diving Decompression Model

B.R. Wienke*

Applied and Computational Physics Division, Los Alamos National Laboratory Los Alamos, NM 87545, USA

Abstract: Linking deep stop model and data, we detail the LANL reduced gradient bubble model (RGBM), dynamical principles, and correlation with the LANL Data Bank. Table, profile, and meter risks are obtained from likelihood analysis, and pertinent applications include nonstop air diving, the Bennett and Maronni 2.5 minute recreational deep stop, C & C Team 450/20 multiple RB dive sequence at 1.4 *atm*, NEDU deep stop tests, and French Navy deep stop profiles. The algorithm enjoys extensive and utilitarian application in mixed gas diving, both in recreational and technical sectors, and forms the bases for released tables, software, and decompression meters used by scientific, commercial, and research divers. The LANL Data Bank is described, and the methods used to deduce risk are detailed. Risk functions for dissolved gas and bubbles are summarized. Parameters that can be used to estimate profile risk are tallied. To fit data, a modified Levenberg-Marquardt routine is employed. The LANL Data Bank presently contains 2879 profiles with 20 cases of DCS across nitrox, trimix, and heliox deep and decompression diving. This work establishes needed correlation between global mixed gas diving, specific bubble model, and deep stop data. The objective is operational diving, not clinical science. The fit of bubble model to deep stop data is chi squared significant to 93%, using the logarithmic likelihood ratio of null set (actual set) to fit set. The RGBM model is thus validated within the LANL Data Bank. Extensive and safe utilization of the model as reflected in user statistics for tables, meters, and software also points to real world validation, that is, one without noted nor reported DCS spikes among RGBM divers. Collecting real world diving data is a global alternative to differential wet and dry testing, a very precise but limited statistical procedure. The approach here for technical, mixed gas, and serious decompression diving parallels the Project Dive Exploration (PDE) effort at DAN for recreational air and nitrox diving, but does not overlap quite obviously. The issue of deep stops versus shallow stops in diving is a hotly debated topic today, and this study reaffirms the efficacy of deep stops, especially as they link naturally to the LANL dual phase bubble model and data. The operational issue of deep stops and staging is one of timing, with questions of time and depth at all stops only addressed within consistent model and ranging data frameworks.

Keywords: Decompression diving, data correlations, model validation, RGBM Data Bank, dual phase models, maximum likelihood.

INTRODUCTION

Within model and data parameters, we outline the LANL reduced gradient bubble model (RGBM), dynamical principles, and correlation with profiles in the LANL Data Bank. Table, meter, and profile risks deduced in likelihood analysis are noted along with risks parameters. Application analyses include the Marroni and Bennett 2.5 *min* recreational deep stop, the C & C 450/20 multiple RB dive sequence at 1.4 *stm*, deep stop tests, and French Navy deep stop profiles. The LANL model enjoys safe, widespread, and utilitarian application in mixed gas diving, both in recreational and technical sectors, and forms the bases of software, released tables and decompression meters used by scientific, commercial, and research divers. Supercomputing power is employed for application and correlation of model and data.

Decompression science and application to diving [1-92] is an ongoing effort. The systematics of gas exchange [11, 14, 39, 45, 59, 79], nucleation [3, 4, 14, 29, 31, 42, 71], bubble growth [7, 11, 28, 66, 71, 90] and elimination [22, 26, 50, 54, 55], counterdiffusion [40, 45, 46, 82, 89], oxygen impact [10, 16, 34, 35, 51, 52, 72], and adaptation [12, 13, 22, 29, 30, 36, 41, 48] upon diving decompression staging [9, 15, 19, 21, 23-27, 30, 34, 36, 39, 44, 49, 57, 61-64, 68-70, 86, 90], and attendant altitude modifications [5, 16, 27, 39, 70, 77] are so complicated that theories only reflect pieces of the puzzle. Computational algorithms, tables, and manned testing are, however, requisite across a spectrum of activities. And the potential of electronic devices to process tables of information or detailed equations underwater is near maturity, with virtually any algorithm amenable to digital implementation. Pressures for even more sophisticated algorithms are expected to grow.

Still computational models enjoy varying degrees of success or failure. More complex models address a greater number of issues, but are harder to codify in decompression tables. Simpler models are easier to codify, but are less comprehensive. Some models are based on first principles, but most are not. Application of models can be subjective in the absence of definitive data, the acquisition of which is tedious, sometimes controversial, and often ambiguous. If deterministic models are abandoned, statistical analysis can address the variability of outcome inherent to random occurrences, but mostly in manner indifferent to specification of controlling mechanisms. The so called dose reponse characteristics of statistical analysis are very

*Address correspondence to this author at the Applied And Computational Physics Division, Los Alamos National Laboratory Los Alamos, N.M. 87545, USA; E-mail: brw@lanl.gov

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attractive in the formulation of risk tables. Applied to decompression sickness incidence, tables of comparative risk offer a means of weighing contributing factors and exposure alternatives.

With quantitative relationships, we underscore the reduced gradient bubble model on dynamical principles, and then its statistical correlations. Both dissolved gas and bubble risk functions are described and parameterized from data in the LANL Data Bank. The RGBM uses a bubble volume to limit exposures, not critical tensions. Bubble volumes are estimates of separated gas phases, and the limit point is called the phase volume. Critical tensions are limit points to dissolved gas buildup in arbitrary tissue compartments, and are often called $M-$ values. The approach is computationally iterative, and though mathematically intensive, diving microprocessors today easily handle calculations in the millisecond processing time frame. The algorithm is the basis of released mixed gas technical tables [NAUI Technical Diving, Tampa, 2002] and simplified recreational air and nitrox tables up to 10, 000 ft elevation. Meter implementations of the RGBM are available and under continuing development, specifically HydroSpace, Zeagle, Steam Machines, Underwater Technologies, Mares, Dacor, Suunto, Plexus, and other players. Commercial RGBM software includes GAP, ABYSS, and HydroSpace EXPLORER Simulator. All have exhibited safe and efficient operation from diving perspectives.

Our intent is to cover aspects of the RGBM not detailed in earlier publications. To this end, we have been collecting mixed gas, deep stop, decompression data in the technical diving arena. This is necessary for model and data correlation, that is, most existing data is based on the shallow stop paradigm required by dissolved gas models, thus predominant versus deep stop data. Deep stop data is valuable, within the RGBM, as well as all other bubble (dual phase) models requiring deep stops algorithmically. While our data is broadbased, we have been able to extract correlation parameters, plus estimate some table, meter, and profile risks. Data collection continues across the gamut of technical, scientific, and research diving.

CONVENTIONS

Note so-called diving units are employed herein, that is, standard SI units for depth and pressure are not used. Pressures and depths are both measured in feet-of-seawater (*fsw*) or meters-of-seawater (*msw*). The conversion is standard,

$$10 \ msw = 33.28 \ fsw = 1 \ atm$$

Breathing mixtures, such as nitrox (nitrogen and oxygen), heliox (helium and oxygen), and trimix (helium, nitrogen, and oxygen), carry standardized notation. If the fraction of oxygen is greater than 21%, the mixture is termed enriched. Enriched nitrox mixtures are denoted EANx, enriched heliox mixtures are denoted EAHx, and enriched trimix mixtures are denoted EATx, for *x* the oxygen percentage. For other mixtures of nitrox and heliox the convention is to name them with inert gas percentage first, and then oxygen percentage, such as, 85/15 nitrox or 85/15 heliox. For trimix, notation is shortened to list the oxygen precentage first, and then only the helium percentage, such as, 15/45 trimix, meaning 15% oxygen, 45% helium, and 40% nitrogen. Air is interchangeably denoted EAN21 or 79/21 nitrox.

Reduced Gradient Bubble Model Synthesis

The RGBM employs a phase volume [38, 85, 90] constraint across the dive profile, tracking excited bubble volumes over the dive. Bubble structures are represented by flexible seed skins with appropriate material properties, permeable to gas diffusion at all pressures and temperatures. Gas diffuses across the bubble interface, and the bubble is subject to Boyle expansion-contraction. The phase volume is an estimate of the cumulative volume of bubbles left at the surface after arbitary depth-time exposures on any diving breathing mixture. Mixtures can be nitrox (oxygen and nitrogen, including air), heliox (oxygen and helium), and trimix (oxygen, helium, and nitrogen). These bubbles can expand and contract during the dive, and are assumed to be excited off an exponential distribution that decreases in number as the radius of the excited bubbles increases. The material properties of these bubbles determine their response to pressure changes, inert gas (nitrogen, helium) diffusion across their interfaces, and the excitation radii for growth. Collectively, material properties are tabulated within equations-of-state (EOS) for lipid and aqueous bubble coatings.

The phase volume constraint equation is written in terms of a phase function, $\dot{\phi}$, varying in time, for τ_{ex} the bubble excitation time, across a distribution of excited bubble seeds, *n*,

$$\int_{\tau_{ex}}^{\tau} \frac{\partial \phi}{\partial t} dt \leq \Phi$$

with, tagging the three bubble processes of excitation, interface gas diffusion, and Boyle expansion-contraction,

$$\dot{\phi} = \frac{\partial \phi}{\partial t}$$

for Φ the separated phase, and τ some (long) cutoff time. More particularly, for Π the total gas tension, taking $\tau \to \infty$, with *V* the separated phase volume, *P* the pressure, and *T* the temperature,

$$\dot{\phi} = \left[\frac{\partial V}{\partial t}\right]_{diffusion} + \left[\frac{\partial V}{\partial t}\right]_{Boyle/Charles} + \left[\frac{\partial V}{\partial t}\right]_{excitation}$$

for,

$$\left[\frac{\partial V}{\partial t}\right]_{diffusion} = 4\pi\beta \exp(\beta\varepsilon) DS \int_{\varepsilon}^{\infty} nr \left[\Pi - P - \frac{2\gamma}{r}\right] dr$$

$$\left[\frac{\partial V}{\partial t}\right]_{Boyle/Charles} = 4\pi\beta \exp(\beta\varepsilon) \int_{\varepsilon}^{\infty} nr^2 [\frac{\partial r}{\partial P}\frac{\partial P}{\partial t} + \frac{\partial r}{\partial T}\frac{\partial T}{\partial t}] dr$$

$$\left[\frac{\partial V}{\partial t}\right]_{excitation} = 4\pi \frac{\partial}{\partial t}\left[\theta(t-\tau_{ex}) \int_{\varepsilon}^{\infty} nr^2 dr\right]$$

and,

$$n = \exp(-\beta r)$$

with quantities as noted, and seed density, n, normalized to the excited phase volume, V,

$$4\pi \int_{\varepsilon}^{\infty} nr^2 dr = exp(-\beta\varepsilon)[8\pi\beta^{-3} + 8\pi\varepsilon\beta^{-2} + 4\pi\varepsilon^2\beta^{-1}] = V$$

for ε the seed excitation radius, r the bubble radius, γ the surface tension, D the diffusivity, S the solubility, and the step (heaviside) function, θ, defined for seed excitation at time, τ_{ex},

$$\theta(t - \tau_{ex}) = 0, \; t \leq \tau_{ex}$$

$$\theta(t - \tau_{ex}) = 1, \; t > \tau_{ex}$$

with the time derivative of the heaviside function a delta function,

$$\frac{\partial\theta(t-\tau_{ex})}{\partial t} = \delta(t-\tau_{ex})$$

Excitation time during a dive occurs whenever the difference between tissue tension and ambient pressure exceeds surface tension by roughly 20% in bubbles of critical radius, r_c, in compartments, τ. It is only the excited phase volume, V, which is tracked throughout the dive, that is, following excitation at time, t_{ex}. In the integrals over time, we do not consider flying-after-diving scenarios here.

In lowest order, number densities of nitrogen and helium bubble seeds are comparable [90]. Experiments suggest that helium bubbles are smaller but more numerous than nitrogen bubble seeds in the same substrate measurements, but differences are small. In zeroth order,

$$n_{He} \simeq n_{N_2} = n$$

In higher order, helium and nitrogen seed densities are averaged over breathing mixture fractions, f_{He} and f_{N_2}, for an effective number density, n,

$$n = \frac{f_{He} n_{He} + f_{N_2} n_{N_2}}{f_{He} + f_{N_2}}$$

The skin equation-of-state (EOS) quantifies the response of bubble films under changes of pressure, P, and temperature, T. An EOS is complicated, often only tabular, or implicitly defined as function of seed volume. To simplify bubble skin EOS lookups, Boyle factors, ξ, are used, so that,

$$\xi PV = nRT$$

as codified in Table **2**. For mixed gas diving, Π is the sum of nitrogen and helium dissolved gas loadings, and the dissolved gradient, G, is,

$$G = \Pi - P$$

Thus the phase function, $\dot{\phi}$, depends on the number of bubbles, n, stimulated into growth by compression-decompression, the supersaturation gradient, G, seed expansion-contraction by radial diffusion, $\partial r/\partial t$, Boyle expansion-contraction with pressure changes, P, and inside temperature, T, in general. The excitation radius, ε, depends on material properties [4, 28, 86-88], and is taken for nitrogen (μm),

$$\varepsilon_{N_2} = 0.007 + 0.016\left[\frac{T}{P}\right]^{1/3} + 0.041\left[\frac{T}{P}\right]^{2/3}$$

and for helium,

$$\varepsilon_{He} = 0.003 + 0.015\left[\frac{T}{P}\right]^{1/3} + 0.025\left[\frac{T}{P}\right]^{2/3}$$

for T measured in absolute ^oK, and P given in fsw, with ranges for virial coefficients, aqueous to lipid material, ξ, s, varying by factors of 0.76 to 4.86 times the values listed above [14, 47]. Both expression above represent fits to RGBM mixed gas data across lipid and aqueous bubble films [4, 60], and are different from other phase models [32, 92]. Values of excitation radii, E, above range from 0.01 to 0.05 μm for sea level down to 500 fsw. This is compared to excitation radii in other models, varying permeability model [90, 91] and tissue bubble diffusion model [32], which vary in the 1 μm range. In the very large pressure limit, excitation radii are in the 1/1, 000 μm range. Table **1** lists excitation radii (air) according to the RGBM.

Table 1. RGBM Excitation Radii

Pressure P (fsw)	Excitation Radius ε (μm)	Pressure P (fsw)	Excitation Radius ε (μm)
13	0.174	153	0.033
33	0.097	183	0.029
53	0.073	283	0.024
73	0.059	383	0.016
93	0.051	483	0.011
113	0.046	583	0.009

Excitation is a continuous and cumulative process, occurring as gas tensions across tissue compartments exceed ambient pressures.

To track Boyle bubble expansion-contraction easily, a set of multipliers, ξ, is tabulated in Table **2** reducing EOS data

Table 2. RGBM Boyle Multipliers

Depth (fsw)	EOS Multiplier ξ
30	0.610
90	0.732
150	0.859
210	0.939
270	1.032
330	1.119
390	1.169
450	1.183
510	1.203

for just pressure changes. For changes in pressure, we have, for bubble assemblies of volume, V, at ambient pressure, P,

$$\xi_i P_i V_i = \xi_f P_f V_f$$

simply, with i and f denoting initial and final states. Multipliers represent a 50/50 lipid-aqueous skin, following Sears [58] and Blank [7]. These multipliers represent a simplification of extensive EOS data for lipid and aqueous materials, condensed into the simpler pressure-volume form above. Obviously, under these multipliers, bubbles are not ideal gases following pressure changes.

To track gas transfer across bubble boundaries, we need mass transport coefficients, DS, for inert gases. Table **3** lists DS for the same 50/50 lipid-aqueous surface, using Frenkel [31], Lango [7], and Batchelor [2]. Mass transfer coefficients are just phenomenological diffusion coefficients for complex gas transport across lipid and aqueous bubble surfaces in tissue and blood. They are a combination of measurements and data extrapolation of gas transfer estimates for inert gases.

Table 3. RGBM Mass Transfer Coefficients

Gas	DS ($\mu m^2/sec\ fsw$)
H_2	72.5×10^{-6}
He	18.4×10^{-6}
Ne	10.1×10^{-6}
N_2	56.9×10^{-6}
Ar	40.7×10^{-6}
O_2	41.3×10^{-6}

Notice that helium has a low mass transport coefficient, some 3 times smaller than nitrogen. Three parameters, closing the set, are nominally,

$$\Phi = 596. \pm 210\ \mu m^3$$

and, for nitrogen and helium,

$$\beta_{N_2} = 0.68 \pm 0.28\ \mu m^{-1}$$

$$\beta_{He} = 0.57 \pm 0.19\ \mu m^{-1}$$

with,

$$2\gamma = \sigma \left[44.7 \left(\frac{P}{T} \right)^{1/4} + 24.3 \left(\frac{P}{T} \right)^{1/2} \right] dyne/cm$$

with material property, σ,

$$0.10 \le \sigma \le 0.85$$

moving from lipid to watery tissue. Later in this analysis, we take $\sigma = 0.5$. The first two parameter sets were obtained from fitting the algorithm to published no decompression time limits (NDLs) for air, nitrox, trimix, and heliox [15, 20, 52, 64, 86]. The third parameter follows from EOS estimates of surface tension, as with excitation radii. Tissues and blood are undersaturated with respect to ambient pressure as far as inert gas partial pressures (tensions). This produces the necessary ingradient for oxygen and outgradient for carbon dioxide in metabolic processes. The difference is termed the inherent undersaturation. The inherent unsaturation (or oxygen window), ψ, takes the form, [39, 90] (fsw),

$$\psi = f_{O_2} P - 2.04(1 - f_{O_2}) - 5.47$$

a linear function of oxygen partial pressure up to 2.0 atm and then constant beyond that, near 70 fsw, with P ambient pressure, and f_{O_2} oxygen fraction. Under compression-decompression, some of this window likely takes up inert gases, denoted, ζ,

$$\zeta = f_{O_2} P - \psi$$

and is added to the inert gas tension. In time, it is assumed, for inert gas, k,

$$\zeta_k = \left[\frac{f_k}{1 - f_{O_2}} \right] \left[f_{O_2} P - \psi \right] \left[1 - exp(-\lambda_k t) \right]$$

for λ_k a decay constant, f_{O_2} again the oxygen fraction, and f_k the inert gas mixture fraction (same across all compartments). Inert gas fractions, f_k, plus oxygen fraction, f_{O_2} sum to 1,

$$f_{O_2} + \sum_{k=1}^{K} f_k = 1$$

where, $K = 2$, $k = N_2$, He, that is, mixed gas diving. Tissue tensions (partial pressures), p_k, for ambient partial pressure, p_{ak}, and initial tissue tension, p_{ik}, evolve in time, t, in usual fashion in compartment, τ_k, according to, given v the (linear) ascent or descent rate between stages,

$$p_k - p_{ak} + \frac{v}{\lambda_k} = vt + \left[p_{ik} - p_{ak} + \frac{v}{\lambda_k} \right] exp(-\lambda_k t) + \zeta_k$$

for,

$$\lambda_k = \frac{0.693}{\tau_k}$$

for τ_k tissue halftime, and ambient pressure, P, as a function of depth, d, in units of fsw,

$$P = \eta d + P_h$$

for surface ambient pressure, P_h,

$$P_h = 3\,3\ exp(-0.0381 h)$$

given h in multiples of 1,000 ft elevation, $\eta = 1$ for salt water, and $\eta = 0.975$ for fresh water. For any gas with mixture fraction, f_k, obviously,

$$p_{ak} = f_k P$$

and total tension, Π, is the sum of component tensions,

$$\Pi = \sum_{k=1}^{K} p_k$$

Nitrogen halftimes, τ_{kN_2} are taken to be 2.5, 5, 10, 20, 40, 80, 120, 180, 240, 320, and 480 min. Helium halftimes, τ_{kHe}, are 2.65 times faster for the same nitrogen compartments,

$$\tau_{kHe} = \frac{\tau_{kN_2}}{2.65}$$

The bubble dynamical protocol in the RGBM algorithm amounts to staging on the seed number averaged, free-dissolved gradient across all tissue compartments, *G*,

$$G\int_{\varepsilon}^{\infty} ndr = (\Pi - P)\int_{\varepsilon}^{\infty} ndr \leq \int_{\varepsilon}^{\infty}\left[\frac{2\gamma}{r}\right]ndr$$

so that,

$$G = (\Pi - P) \leq \beta\, exp\,(\beta\varepsilon)\int_{\varepsilon}^{\infty} exp\,(-\beta r)\left[\frac{2\gamma}{r}\right]dr$$

for ε the excitation radius at *P* and *T*. Time spent at each stop is iteratively calculated so that the total separated phase, Φ, is maintained at, or below, its limit point. This requires some computing power, but is attainable in diver wrist computers presently marketed commercially. Stops are computed in 10 *fsw* increments. An important feature of the iterative process is noted:

1. separated phase volume, Φ, is the same for all inert gases;
2. the gradient, *G*, is slowly varying as seeds are excited into growth, expand or contract as gas diffuses across bubble films, and expand or contract as ambient pressure changes.

The combination of the two produces dramatically different staging regimens than classical dissolved gas protocols. This (new) staging protocol has been in use for the past 8 -12 years, data is being collected from divers, and the process of evaluation and updating is a continuous one.

LANL Profile Data Bank

Divers using bubble models are reporting their profiles to a Data Bank, located at LANL (also NAUI Technical Diving Operations). The profile information requested is simple:

1. bottom mix/pp_{O_2} depth, and time (square wave equivalent);
2. ascent and descent rates;
3. stage and decompression mix/pp_{O_2} depths, and times;
4. surface intervals;
5. time to fly;
6. diver age, weight, and sex;
7. outcome (health problems), rated 1 -5 in order of poor (DCS) to well.

This information aids validation and extension of model application space. Some 2, 879 profiles now reside in the LANL Data Bank. There are 20 cases of DCS in the data file. The underlying DCS incidence rate is, p =20/2879= 0.0069, below 1%. Stored profiles range from 150 *fsw* down to 840 *fsw*, with the majority above 350 *fsw*. All data enters through the author (BRW), that is, divers, profiles, and outcomes are filtered. A summary breakdown of DCS hit (bends) data consists of the following:

1. OC deep nitrox reverse profiles –5hits (3DCS I, 2DCS II)
2. OC deep nitrox – 3 hits (2 DCS I, 1 DCS II)
3. OC deep trimix reverse profiles – 2 hits (1 DCS II, 1 DCS III)
4. OC deep trimix – 2 hits (1 DCS I, 1 DCS III)
5. OC deep heliox – 2 hits (2 DCS II)
6. RB deep nitrox –2hits (1DCS I, 1DCS II)
7. RB deep trimix –2hits (1DCS I, 1DCS III)
8. RB deep heliox – 2 hits (1 DCS I, 1 DCS II)

DCS I means limb bends, DCS II implies central nervous system (CNS) bends, and DCS III denotes inner ear bends (occurring mainly on helium mixtures). Both DCS II and DCS III are fairly serious afflictions, while DCS I is less traumatic. Deep nitrox means a range beyond 150 *fsw*, deep trimix means a range beyond 200 *fsw*, and deep heliox means a range beyond 250 *fsw* as a rough categorization. The abbreviation OC denotes open circuit, while RB denotes rebreather. Reverse profiles are any sequence of dives in which the present dive is deeper than the previous dive. Nitrox means an oxygen enriched nitrogen mixture (including air), trimix denotes a breathing mixture of nitrogen, helium, oxygen, and heliox is a breathing mixture of helium and oxygen. None of the trimix nor heliox cases involved oxygen enriched mixtures on OC, and RB hits did not involve elevated oxygen partial pressures above 1.4 *atm*. Nitrogen-to-helium (*heavy − to − light*) gas switches occured in 4 cases, violating contemporary ICD (isobaric counterdiffusion) protocols [35, 40, 44-46]. Isobaric counter-diffusion refers to two inert gases (usually nitrogen and helium) moving in opposite directions in tissues and blood. When summed, total gas tensions (partial pressures) can lead to increased super-saturation and bubble formation probability. None of the set exhibited full body nor CNS (central nervous system) oxygen toxicity (popularly called *oxtox*). The 20 cases come after the fact, that is diver distress with hyperbaric chamber treatment following distress. The Appendix describes many of the profiles in the LANL Data Bank, as well as broader field testing reported to us. Profiles come from seasoned divers using wrist slate decompression tables with computer backups. Some profiles come to us directly as computer downloads, which we transcribe to the requisite format.

Profiles come from the technical diving community at large, essentially mixed gas, extended range, decompression, and extreme diving. Profiles from the recreational community are not included, unless they involve extreme exposures on air or nitrox (many repetitve dives, deeper than 150 *fsw*, altitude exposures, etc). This low rate makes statistical analysis difficult, and we use a global approach to defining risk after we fit the model to the data using maximum likelihood. The maximum likelihood fit links directly to the binomial probability structure of DCS incidence in divers and aviators. Consider it briefly, and the likelihood maximization technique [8, 43, 53].

Probabilistics

Decompression sickness is a hit, or no hit, situation. Statistics are binary, as in coin tossing. Probabilities of occurrence are determined from the binomial distribution, which measures the numbers of possibilities of occurrence and non-occurrence in any number of events, given the incidence rate. Specifically, the probability, P, in a random sample of size, N, for n occurrences of decompression sickness and m non-occurrences, takes the form,

$$P(n) = \frac{N!}{n!\,m!}\,p^n q^m$$

with,

$$n + m = N$$

p the underlying incidence rate (average number of cases of decompression sickness), and q,

$$q = 1 - p$$

the underlying nonincidence. For large sample sizes, $N = n + m$,

$$ln\ P(n) \approx N\ l\ nN - n\ ln\ n - m\ ln\ m + n\ ln\ p + m\ ln\ q$$

The likelihood of binomial outcome, Φ, of N trials is the product of individual measures of the form,

$$\Phi(n) = p^n q^m = p^n (1 - p)^m$$

given n cases of decompression sickness and m cases without decompression sickness, and,

$$n + m = N$$

The natural logarithm of the likelihood (LL), Ψ, is easier to use in applications, and takes the form,

$$\Psi = \ln \Phi = n \ln p + m \ln (1 - p)$$

and is maximized when,

$$\frac{\partial \Psi}{\partial p} = 0$$

The multivalued probability functions, $p(x)$, generalize in the maximization process according to,

$$\frac{\partial \Psi}{\partial p} = \sum_{k=1}^{K} \frac{\partial \Psi}{\partial x_k} \frac{\partial x_k}{\partial p} = 0$$

satisfied when,

$$\frac{\partial \Psi}{\partial x_k} = 0 \quad for\ k = 1, K$$

In application, such constraints are most easily solved on computers, with analytical or numerical methods. For RGBM analysis, the likelihood, Ψ, is typically a function of 2 -3 parameters over the whole set of profiles. This requires extensive computing power coupled to sophisticated numerical techniques and software.

Hundreds of air dives were analyzed using this procedure, permitting construction of decompression schedules with 95% and 99% nonincidence (5% and 1% bends incidence). Tables were published by US Navy investigators [5, 73], and Table **4** tabulates the corresponding nonstop time limits (p =0.05, 0.01), and also includes the standard US Navy (Workman) limits [21, 23, 86] for comparison. Later re-evaluations of the standard set of nonstop time limits estimate a probability rate of 1.25% for the limits. In practice, incidence rates are below 0.001%, and most divers do not dive to the limits.

Table 4. Nonstop Time Limits for 1% and 5% DCS Probability

Depth ***d*** **(*fsw*)**	**Nonstop Limit** t_n **(*min*)** ***p*** **=.05**	**Nonstop Limit** t_n **(*min*)** ***p*** **=.01**	**Nonstop Limit** t_n **(*min*)** **US Navy**
30	240	170	
40	170	100	200
50	120	70	100
60	80	40	60
70	80	25	50
80	60	15	40
90	50	10	30
100	50	8	25
110	40	5	20
120	40	5	15
130	30	5	10

LANL Data Correlations and Risk Estimators

To perform risk analysis with the LANL Data Bank, an estimator need be selected. For diving, dissolved gas and phase estimators are useful. Two, detailed earlier, are extended here. First is the dissolved gas super saturation ratio, historically coupled to Haldane models, ρ, written in modified ratio form,

$$\rho(\kappa,\lambda,t) = \kappa\left[\frac{\Pi(t) - P(t)}{P(t)}\right] - \kappa\ exp\,(-\lambda t)$$

and second, ψ, is the separated bubble volume, invoked by dual phase models,

$$\psi(\gamma,\mu,t) = \gamma\left[\frac{\phi(t)}{\phi_i(t)}\right] - \gamma\ exp\,(-\mu t)$$

with $\phi(t)$ the bubble volume due to excitation, diffusion, and Boyle expansion-contraction, and ϕ_i the initial bubble excitation volume. The exponential terms in both risk functions merely insure data smoothing for short dives, that is, as $t \rightarrow 0$, then $r \rightarrow 0$, too. For long dives, $t \rightarrow \infty$, the exponential terms vanish. Physically, the exponential terms also link to bubble extinction, not discussed herein. Both risk functions vary in time, exposure, and staging. For simplicity, the asymptotic exposure limit is used in the likelihood integrals for both risk functions, r, across all compartments, τ,

$$1 - r(\kappa,\lambda) = exp\left[-\int_0^\infty \rho(\kappa,\lambda,t)dt\right]$$

$$1 - r(\gamma,\mu) = exp\left[-\int_0^\infty \psi(\gamma,\mu,t)dt\right]$$

with *hit* −*no hit*, likelihood function, Ω, of form,

$$\Omega = \prod_{k=1}^{K} \Omega_k$$

and logarithmic reduction, Ψ,

$$\Psi = ln\ \Omega$$

where, $\delta_k = 0$ if DCS does not occur in profile, k, or, $\delta_k = 1$ if DCS does occur in profile, k.To estimate κ, λ, γ, and μ in maximum likelihood, a modified Levenberg-Marquardt [6, 43] algorithm is employed (*SNLSE*, Common Los Alamos Applied Mathematical Software Library) [84], a nonlinear least squares data fit (NLLS) to an arbitrary logarithmic function (minimization of variance over K data points with $L2$ error norm). The same technique was applied to estimating separated phase volume and inert gas densities.

We assign numerical tasks to processors on the LANL Blue Mountain Machine, a massively parallel processor (MPP) with 2, 000 nodes according to:

1. each tissue compartment, τ, then, within each compartment;
2. only nitrox data points;
3. only trimix data points;
4. only heliox data points;
5. both nitrox and trimix data points;
6. both nitrox and helium data points;
7. both heliox and trimix data points;
8. all heliox, nitrox, and trimix data points.

estimating κ, λ, γ, and μ across all domains. The last case, all data, is the full set employed in risk analysis, but there wasn't much difference in the estimators, seen in mean error estimates across the partitioned data structures. For 11 tissue compartments, and 7 data sets, 77 risk estimates emerge. Only maximum tissue risks are finally averaged and variance computed. In diver staging, certain tissue compartments control the exposure, This is true within dissolved gas algorithms, as well as bubble algorithms. Finally, we find across the partioned data structures, 2-8 above:

$$\kappa = 0.91 \pm 0.14\ min^{-1}$$

$$\lambda = 0.28 \pm 0.11\ min^{-1}.$$

and, similarly.

$$\gamma = 0.09 \pm 0.07\ min^{-1}$$

$$\mu = 0.88 \pm 0.46\ min^{-1}$$

For notational shorthand, we abbreviate supersaturation and bubble risk functions,

$$\sigma = r(\kappa, \lambda)$$

$$\beta = r(\gamma, \mu)$$

Logarithmic Likelihood and Significance

The data is relatively coarse grained, making compact statistics difficult. The incidence rate across the whole set is small, on the order of 1% and smaller. Fine graining into depths is not meaningful yet, so we breakout data into gas categories (nitrox, heliox, trimix), as tabulated earlier. Table **5a** indicates the breakdown.

The DCS hit rate with nitrox is higher, but not statistically meaningful across this sparse set. The last entry is all mixes, as noted previously. In the above set, there are 35 *marginals*.

Table 5a. Profile Data

Mix	Total Profiles	DCS Hits	Incidence
OC nitrox	344	8	0.0232
RB nitrox	550	2	0.0017
all nitrox	894	10	0.0112
OC trimix	656	4	0.0061
RB trimix	754	2	0.0027
all trimix	1410	6	0.0042
OC heliox	116	2	0.0172
RB heliox	459	2	0.0044
all heliox	575	4	0.0070
all	2879	20	0.0069

The logarithmic likelihood (LL), Ψ, is a rough metric for fits to bubble and supersaturation risk estimators. The canonical value, Ψ_6, is the LL for the 6 OC/RB gas control data set. No fit value, Ψ, will better the canonical value, Ψ_6, that is,

$$\Psi_6 = -112.9$$

$$\Psi \le \Psi_6$$

meaning all fits will be more negative (smaller LL). Results are tabulated as follow in Table **5b**.

Table 5b. Logarithmic Likelihood and Logarithmic Likelihood Ratio

Estimator	LL	Parameters	LLR	α
6 step set	$\Psi_6 = 112.9$	p = 0.0232, 0.0061, 0.0172,		
		0.0036, 0.0027, 0.0044		
3 step set	$\Psi_3 = -118.4$	p = 0.0112, 0.0042, 0.0079	$\Gamma_3 = 11.0$	0.013
1 step set	$\Psi_1 = -119.2$	p =0.0069	$\Gamma_1 = 12.6$	0.033
σ	$\Psi_{sat} = -210.6$	$\kappa = 0.91 \pm 0.14\ min^{-1}$	$\Gamma_{sat} = 92.2$	0.001
		$\lambda = 0.28 \pm 0.11\ \mathrm{min}^{-1}$		
β	$\Psi_{bub} = -113.3$	$\gamma = 0.09 \pm 0.07\ min^{-1}$	$\Gamma_{bub} = 0.8$	0.933
		$\mu = 0.88 \pm 0.46\ \mathrm{min}^{-1}$		

The logarithmic likelihood ratio (LLR), denoted Γ, tests two models, and is χ^2 distributed,

$$\Gamma = 2(\Psi_6 - \Psi)$$

for Ψ the bubble and supersaturation estimators in Table **5b**. The percentage point, α, is the area under the χ^2 curve, from $\chi^2 = \Gamma$ to ∞,

$$\int_{\chi^2_{\alpha,\nu}}^{\infty} \chi^2(x,\nu)dx = \alpha$$

for ν the degrees of freedom (6 -the number of bubble, supersaturation, 3 step, or 1 step degrees of freedom). The *hit − no hit* criteria for the bubble estimator is the phase volume, Φ, while standard *M −values* are the criteria for the supersaturation estimator. Deep stops clobber traditional *M − values*.

Clearly, the supersaturation risk function does not correlate well, compared to the bubble risk function. It does not work here in the deep decompression arena, but others [73, 74] have shown it correlates in the nonstop, or light decompression, limits. In those limits, bubble models and supersaturation models tend to converge, simply because phase growth is minimal.

Capsule Summary

The LANL reduced gradient bubble model (RGBM) has been detailed, including correlations and data linkage within the LANL Data Bank. The Bank stores technical, mixed gas diving profiles with outcomes. Some 2800+ deep stop profiles reside within the Bank, with 20 cases of DCS. Parameters within the RGBM have been extracted from the LANL Data Bank using maximum likelihood techniques, and a Monte Carlo-like sampling technique was employed to accelerate likelihood analysis. Risk estimates for select NDLs, tables, meter algorithms, tests, and diver profiles in the LANL Data Bank were tabulated, using a bubble phase volume estimator integrated over the whole profile. Model, data, and operational diving are congruent, providing a useful and safe platform. Widespread usage statistics for meters, tables, software, and Training Agency protocols underscored safe and consistent application of the LANL model across diverse sectors. All of the above represent ongoing testing and validation efforts which surpass scattered clinical tests, wet and dry, numberwise.

In addition to the gas transport comparison of dissolved gas staging versus bubble staging, analysis suggests broadly:

1. deep stop data is intrinsically different from data collected in the past for diving validation, in that previous data is mainly based on shallow stop diver staging, a possible bias in dive planning;
2. deep stop data and shallow stop data yield the same risk estimates for nominal, shallow, and nonstop diving because bubble models and dissolved gas models converge in the limit of very small phase separation;
3. if shallow stop data is employed in all cases covered, dissolved gas risk estimates will be usually higher than those computed herein;
4. bubble risks estimated herein are higher than risk estimates in other analyses, perhaps a conservative bias;
5. data entry in the LANL Data Bank is a ongoing process of profile addition, extended exposure-depth range, and mixed gas diving application.

Data specifically underscores technical diving trends:

1. pure O_2 or *EAN*80 are standard OC switch gases in the 20 *fsw* zone;
2. deep stops are standard across mixed gas diving, and DCS spikes are nonexistent;
3. deep switches to nitrogen mixes off helium mixes are avoided by technical divers, instead oxygen fraction is increased by decrease in helium fraction;
4. deep stop dive computers serve mostly as backup or bailout, with tables and dive planning software the choice for deep stop diving;
5. DCS spikes across mixed gas, decompression, and deep stop diving are non existent using deep stop tables, meters, and software;
6. DCS incidence rates are higher for technical diving versus recreational diving, but still small;
7. RB usage is increasing across diving sectors;
8. wrist dive computers possess chip speeds that allow full resolution of even the most extensive bubble models;
9. nitrox diving in the recreational sector is exploding;
10. technical diving data is most important for correlating models and data;
11. technical divers do not dive air, particularly deep air, with trimix and heliox the choices for deep excursions;
12. released deep stop tables, software, and meters enjoy extensive and safe utility among professional divers;
13. technical diving is growing in leaps and bounds, with corresponding data accessible off computers and bottom timers;
14. more cross talk across military, scientific, research, exploration, and commercial diving is desirable.

This work establishes needed correlation between global mixed gas diving, specific bubble model, and deep stop data. The objective is operational diving, not clinical science. The operational issue of deep stops and staging is one of timing, with questions of time and depth at all stops only addressed within consistent model and ranging data frameworks. To that end, we find deep stops are not riskier than shallow stops, that both can accomplish the same end, and that deep stops are more efficient timewise than shallow stops.

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providing user statistics and data on RGBM training regimens, tables, and meter implementations.

LIST OF SYMBOLS

Φ = Separated bubble volume

t = Time

ϕ = Time rate of change of separated bubble volume

V = Bubble volume

β = Bubble normalization constant

ε = Bubble excitation radius

r = Bubble radius

γ = Bubble surface tension

D = Inert gas diffusivity

S = Inert gas solubility

Π = Total inert gas tension

P = Ambient pressure

T = Ambient temperature

n = Bubble number density

f = Gas number fraction

G = Dissolved gas gradient

ξ = EOS multiplier

Φ = Bubble volume limit point

σ = Lipid-aqueous weighting fraction

ψ = Oxygen window

ζ = Inert gas window fraction

p = Inert gas tension

τ = Tissue halftime

λ = Tissue rate constant

η = Specific density relative to seawater

h = Altitude in multiples of 1, 000 ft

$P(n)$ = Binomial probability

$\phi(n)$ = Binomial outcome

Ψ = Logarithmic likelihood

ρ = Supersaturation risk function

ϕ = Bubble risk function

Ω = Likelihood hit function

Γ = Logarithmic likelihhod ratio

χ^2 = Chi squared distribution

APPENDIX A

Applications

This analysis suggests that deep stops are both safe and compact statistically for the LANL model and set. Coupled gas transport analysis suggests that deep stops and shallow stops can both be staged safely, but deep stops are more efficient in controlling bubble growth and are usually shorter in overall dive time duration. This is important in deep decompression diving, but also affects recreational air diving as well. Before considering deep stops, let's take a look at risk estimates for recreational air diving out to the no decompression time limits (NDLs).

Nonstop and Repetitive Air Diving

Nonstop limits (NDLs), denoted t_{nn}, from the US Navy, PADI, NAUI, and ZHL (Buhlmann) Tables [15, 16] provide a set for comparison of relative DCS risk. Listed in Table **6** are the NDLs and corresponding risks for the nonstop excursion, assuming ascent and descent rates of 60 *fsw/min* (no safety nor deep stops). Dissolved gas and phase risk estimates vary little for cases, and only the phase estimates are included. Surface intervals (SIs) between dives are time spent at the surface.

Table 6. Risk Estimates for Standard Air NDLs

USN NDL		Risk	PADI NDL	Risk	NAUI NDL	Risk	ZHL NDL	Risk
***d* (*fsw*)**	***tn* (*min*)**	***β***	***tn* (*min*)**	***β***	***tn* (*min*)**	***β***	***tn* (*min*)**	***β***
35	310	4.3%	205	2.0%			181	1.3%
40	200	3.1%	140	1.5%	130	1.4%	137	1.5%
50	100	2.1%	80	1.1%	80	1.1%	80	1.1%
60	60	1.7%	55	1.4%	55	1.4%	57	1.5%
70	50	2.0%	40	1.2%	45	1.3%	40	1.2%
80	40	2.1%	30	1.3%	35	1.5%	30	1.3%
90	30	2.1%	25	1.5%	25	1.5%	24	1.4%
100	25	2.1%	20	1.3%	22	1.4%	19	1.2%
110	20	2.2%	13	1.1%	15	1.2%	16	1.3%
120	15	2.0%	13	1.3%	12	1.2%	13	1.3%
130	10	1.7%	10	1.7%	8	1.3%	10	1.7%

Table 7. Dissolved and Separated Phase Risk Estimates for Nominal Profiles

Profile (*depth/time*)	Descent Rate (*msw/min*)	Ascent Rate (*msw/min*)	Safety Stop (*depth/time*)	Risk β	Risk σ
14 *msw*/38 *min*	18	9	5 *msw*/3 *min*	0.0034	0.0062
19 *msw*/38 *min*	18	9	5 *msw*/3 *min*	0.0095	0.0110
28 *msw*/32 *min*	18	9		0.0200	0.0213
37 *msw*/17 *min*	18	9	5 *msw*/3 *min*	0.0165	0.0151
18 *msw*/31 *min*	18	9	5 *msw*/3 *min*	0.0063	0.0072
	18	9		0.0088	0.0084
	18	18		0.0101	0.0135
	18	18	5 *msw*/3 *min*	0.0069	0.0084
17 *msw*/32 *min*	18	9	5 *msw*/3 *min*		
SI 176 *min*					
13 *msw*/37 *min*	18	9	5 *msw*/3 *min*		
SI 174 *min*					
23 *msw*/17 *min*	18	18	5 *msw*/3 *min*	0.0127	0.0232

Risks are internally consistent across NDLs at each depth, and agree with the US Navy assessments in Table 4. Greatest underlying risks occur in the USN shallow exposures. The PADI, NAUI, and ZHL risks are all less than 2% for this set, and risks for single DCS incidence are less than 0.02. PADI and NAUI have reported that incidence rates (p) across all exposures are less than 0.001%, so considering their enviable track record of diving safety, our estimates are liberal. ZHL risk estimates track as the PADI and NAUI risks, again, very safely. Estimates were corroborated [Gerth, priv comm, 2001] within data sets at Duke both in Tables **6** and **7**.

Next, the analysis is extended to profiles with varying ascent and descent rates, safety stops, and repetitive sequence [53, 78-80, 82]. Table **7** lists nominal profiles (recreational) for various depths, exposure and travel times, and safety stops at 5 *msw*. Mean DCS estimates, r, are tabulated for both dissolved gas supersaturation ratio and excited bubble volume risk functions, with nominal variance, $r_{\pm} = r \pm 0,004$, across all profiles.

The ZHL (Buhlmann) NDLs and staging regimens are widespread across decompression meters presently, and are good representations for dissolved gas risk analysis. The RGBM is newer, more modern, and is coming online in decometers and associated software. For recreational exposures, the RGBM collapses to a dissolved gas algorithm. This is reflected in the risk estimates above, where estimates for both models differ little [9, 20, 23, 36, 86].

Simple comments hold for the analyzed profile risks. The maximum relative risk is 0.0232 for the 3 dive repetitive sequence according to the dissolved risk estimator. This translates to 2% profile risk, which is comparable to the maximum NDL risk for the PADI, NAUI, and ZHL NDLs. This type of dive profile is common, practiced daily on liveaboards, and benign. According to Gilliam, the absolute incidence rate [75] for this type of diving is less than 0.02%. Again, our analyses overestimate risk. Effects of slower ascent rates and safety stops are seen only at the 0.25% to 0.5% level in relative surfacing risk. Safety stops at 5 *msw* for 3 *min* lower relative risk an average of 0.3%, while reducing the ascent rate from 18 *msw/min* to 9 *msw/min* reduces relative risk an average of 0.35%. Staging, NDLs, and contraints imposed by decometer algorithms are consistent with acceptable and safe recreational diving protocols. Estimated absolute risk associated across all ZHL NDLs and staging regimens analyzed herein is less than 2.32%, probably much less in actual practice. That is, we use $p = 0.0069$, and much evidence suggests $p < 0.0001$, some ten times safer. While powerful, statistical methods are neither deterministic nor mechanistic, and cannot predict on first principles. But as a means to table fabrication with quoted risk, such approaches offer attractive pathways for analysis.

Questions of what risk is acceptable to the diver vary. Sport and research divers would probably opt for small risk (1% or less), while military and commercial divers might live with higher risk (5%), considering the nearness of medical attention in general. Many factors influence these two populations, but fitness and acclimatization would probably play strategically. Recent Doppler and wet tests are interesting, including our recorded CCR 16 dive sequence to 450 *fsw*. Gas transport [13, 84] analysis of these applications follows, along with bubble risk estimates.

Bennett and Maronni 2.5 Minute Recreational Deep Stop

Deep stops are already mainliners in some training agency protocols for no and light decompression diving on air and nitrox. The prescription is to make a deep stop at half depth for 1 -3 *min*, followed by a shallow stop in the 15 *fsw* zone for 1 -2 *min*. In Table **8a**, we cite bubble surfacing risks for a deep stop at half depth for 1 *min*, 2.5 *min*, and 4 *min*, the middle case suggested by Bennett and Maronni from Doppler scoring [Bennett, priv comm, 2008], followed by direct ascent to the surface. Surfacing supersaturation risks

Table 8a. Comparative Bubble Risks for Recreational Deep Stops

Depth (*fsw*)	Time (*min*)	No Stop β	1 *min* Stop β	2.5 *min* Stop β	4 *min* Stop β
80	40	2.10%	1.93%	1.90%	1.91%
90	30	2.10%	1.87%	1.83%	1.84%
100	25	2.10%	1.74%	1.71%	1.72%
110	20	2.20%	1.65%	1.61%	1.62%
120	15	2.00%	1.50%	1.46%	1.47%
130	10	1.70%	1.29%	1.25%	1.26%

are tabulated in Table 8b for comparison. Dives are carried out to the (old) US Navy NDLs for easy reference. Deep stops for less than 2.5 *min* reduce recreational risk out to the Navy NDLs in all cases. Bubble risks decrease for short deep stops and then increase as stop times increase. As stop times continue to increase, the dives will require decompression. In other words, with increasing deep stop time, the dives become multilevel decompression dives. Obviously, the playoff of deep stop time against bottom time is a minimax problem. This is traced to bubble behavior with increased gas tensions for increasing deep stop time. In all cases, stop time in the shallow zone was 1 *min*. Longer stop times in the shallow zone had little effect on surfacing risks. Shallow stops probably serve better to teach buoyancy control to neophytes.

Ascent and descent rates were standard in the analysis, that is, 30 *fsw/min* and 60 *fsw/min* respectively. The small risk spread for 1 -4 *min* accommodates recreational deep stop training regimens, that is, 1 -3 *min* deep half stop for many agencies.

Corresponding supersaturation risks in Table **8b** are seen to increase montonically with length of deep stop. This is to be expected in dissolved gas models, with exposures at increasing depths for increasing times cascading tissue tensions, oblivious to any bubble-dissolved gas interactions.

C & C Team 450/20 Multiple RB Dive Sequence at 1.4 *atm*

Details of a 16 dive sequence by members of the C & C Team to 450 *fsw* for 20 *min* at 1.4 *atm* follow. Dives were successfully completed in tandem without mishap, and are included in the LANL Data Bank. All dives follow the same schedule, as given in Table **9**. Oxtox (both CNS and full body) metrics are included. Diver Tags and Outcomes are tabulated, according to the LANL Data Bank profile schema described previously. Diver Tag 1 is one of the authors (BRW). Risk estimates (both bubble and supersaturation) are noted, along with binomial probabilities for 16 tandem dives within a LANL Data Bank underlying incidence rate of 0.69%. Four additional dives in the same sequence were also performed without mishap, but are not included because of larger fluctuations about 450 *fsw*. Bottom fluctuations in the 16 dive sequence were ±5 *fsw* maximum for longer than a minute.

Diluent is 10/80 trimix with a pp_{O_2} setpoint of 1.4 *atm*. The cumulative CNS clock fractions exceed a (traditional) limit of 1.0, while OTU uptake remains below a (traditional) limit of 650 *min*. There is likely greater variability in oxtox limit points than decompression limit points. Descent and ascent rates are standard, except in the 30 *fsw* zone where the ascent rate is 1 *fsw/min*. The binomial probability of no hits is P (0), while the probability of 1 hit is P (1). The probability of 2 or more hits is vanishingly small for underlying incidence of 0.69%.

NEDU Deep Stop Air Tests

The Navy Experimental Diving Unit recently tested their version of air deep stops [NEDU, priv comm, 2007] with a rejection DCS rate. Profiles tested are given in Table **10**, along with a suggested LANL deep stop profile. Profile NEDU 1 incurred a 5.5% DCS hit rate, while NEDU 2, incurred a lower 1.5% DCS hit rate.

Table 8b. Comparative Supersaturation Risks for Recreational Deep Stops

Depth (*fsw*)	Time (*min*)	No Stop σ	1 *min* Stop σ	2.5 *min* Stop σ	4 *min* Stop σ
0	40	2.10%	2.12%	2.18%	2.26%
90	30	2.10%	2.13%	2.20%	2.29%
100	25	2.10%	2.15%	2.23%	2.34%
110	20	2.20%	2.24%	2.32%	2.41%
120	15	2.00%	2.10%	2.20%	2.38%
130	10	1.70%	1.78%	1.91%	2.13%

Table 9. RB 16 Dive Sequence at 1.4 *atm*

Dive Tags = 2042 -2058
Diver Tags = 3, 20, 5, 1, 9, 6, 10, 2, 14, 4, 15, 7, 8, 11, 16, 12
Diver Outcomes = 3, 4, 3, 3, 4, 3, 4, 3, 4, 3, 3, 3, 4, 3, 4, 3
Underlying Incidence = 20/2879

Depth (*fsw*)	Time (*min*)	CNS Clock (*fraction*)	OTU Uptake (*min*)
450	20	.17	32.6
360	0.5	.01	0.8
350	0.5	.01	0.8
340	0.5	.01	0.8
330	0.5	.01	0.8
320	0.5	.01	0.8
310	0.5	.01	0.8
300	1.0	.02	1.6
290	1.0	.02	1.6
280	1.0	.02	1.6
270	1.0	.02	1.6
260	1.0	.02	1.6
250	1.0	.02	1.6
240	1.0	.02	1.6
230	1.5	.03	1.8
220	1.5	.03	1.8
210	2.0	.03	4.1
200	2.0	.03	4.1
190	2.0	.03	4.1
180	2.0	.03	4.0
170	2.0	.02	4.0
160	2.5	.02	4.0
150	2.5	.02	3.9
140	3.5	.03	5.7
130	5.0	.05	9.0
120	5.0	.04	8.5
110	5.0	.04	8.4
100	5.5	.04	9.0
90	6.0	.05	9.8
80	8.0	.07	13.0
70	8.0	.07	12.5
60	9.5	.08	15.5
50	11.0	.10	17.9
40	12.0	.10	19.5
30	8.5	.07	13.8
20	10.5	.09	17.1
10	17.0	.11	25.2
	211.5	1.38	262.2
,	β =4.27%,	σ =12.67%	
	P (0) = 89.4%,	P (1) = 10.4%	

so computed bubble risk, β, is below the binomial probability, P (1).

Table 10. Comparative NEDU Air Deep Stop Schedules

Depth (*fsw*)	NEDU 1 Time (*min*)	NEDU 2 Time (*min*)	LANL Time (*min*)
170	30	30	30
120			0.5
110			1.5
100			2.5
90			3.5
80			4.5
70			5.0
70	12		5.0
60	17		7.0
50	15		11.0
40	18	9	14.5
30	23	23	22.0
20	17	52	28.5
10	72	93	59.9
	206	207	195
σ	5.6%	2.4%	3.4%
β	10.6%	3.2%	2.6%

Bubble risk is higher in both NEDU 1 and NEDU 2, but large in NEDU 1. NEDU 1 is a multilevel decompression dive with inadequate treatment in the shallow zone. Initial deep stops in NEDU 1 did not control bubble growth, and the length of the stay in 70, 60, and 50 *fsw* builds up dissolved gas in the middle range tissues, which then diffuses into bubbles causing them to grow. NEDU 2 is classic with no deep stops, and very long times in the shallow zone to effect decompression. The LANL schedule has deeper stops, shorter midzone times, and then shorter times in the shallow zone compared to both NEDU 1 and NEDU 2. One important factor here is the shape of the decompression schedule, that is the LANL profile is shorter overall, with NEDU 1 and NEDU 2 profiles exhibiting supersaturation staging with shallow belly and tail, while the LANL profile is steeper exhibiting bubble staging with deeper stops and steeper ascent rate. Both NEDU profiles are not of the genre typically dived by users of modern deep stop tables, software, and meters.

Gas transport [13, 84] analyses on both NEDU schedules suggests that NEDU 1 produces 15% -30% larger bubble volumes on surfacing, due to the longer stay in the mid zone, while NEDU 2 produces surfacing bubble volumes 3% -5% larger than surfacing bubble volumes in the LANL profile. Surfacing bubble volumes in the LANL profile were close to the staging limit point.

French Navy Deep Stop Schedules

The French Navy also tested deep stop air schedules [Blatteau, priv comm, 2008]. Three protocols on deep air were employed and none exhibited Grade 4 Doppler bubbles. Analysis centered on just Grade 3 bubbles. For purposes of deep stop analysis, Protocol 1, a dive similar to NEDU 1, is interesting. Protocol 1 is a deep air dive to 200 fs*w* for 20 min, with ascent staging according to Table **11**. Contrasting staging strategies are denoted MN90, the standard French Navy dissolved gas regimen, and LANL. Outside of World Navies, few diving sectors today even contemplate air decompression diving to 200 fsw. Risks in air dives beyond 150 fs*w* are known to increase by factors of 10 over similar dives at shallower depth [65, 86]. This is, of course, one major reason why trimix and heliox become mixtures of choice for deep and decompression diving worldwide, across commercial, scientific, exploration, and research sectors.

Table 11. French Navy Air Deep Stop Schedules at 200 *fsw*

Ascent Rate *fsw/min* Starting at 90 *fsw* Depth (*fsw*)	Protocol 1 10 Time (*min*)	MN90 20 Time (*min*)	LANL 30 Time (*min*)
200	20	20	20
130			0.5
120			0.5
110			1.0
100			1.0
90			1.0
80	1		1.5
70	1		2.0
60	2		2.0
50	2		2.5
40	4		3.0
30	6	3	6.0
20	9	8	7.0
10	22	32	8.0
	78	68	65
β	3.9%	2.2%	2.1%

By contrast, LANL staging starts deeper, is shorter overall, and has smaller bubble risk than Protocol 1. Protocol 1, however, tracks more closely with LANL than NEDU 1, and exhibits lower risk than NEDU 1. However, run time for Protocol 1 versus MN90 is longer, unlike conventional bubble model run times. Estimated bubble risks, β, are tabulated at the bottom of Table **11**.

Gas Transport Analysis

With regard to the preceeding dives and schedules, a couple of points are interesting. These follow from a closer look at dissolved and bubble gas phases across the profiles, using LANL tools and selected way points on the dives. These comments also apply to deep and decompression staging using traditional dissolved gas models and tables [13, 78, 89]. Remember these comments are made within the LANL model framework and attendant data correlation:

1. bubble growth in the deep zone of decompression profiles NEDU 1 and Protocol 1 is not constrained in their version of deep stop air tests;
2. deep stops are not deep enough in NEDU 1 and Protocol 1, nor are follow stops;
3. critical phase volume limit points are exceeded in NEDU 1 and Protocol 1 even before the diver exits, in other words, along the decompression glide path underwater;
4. the recreational 2.5 *minute* stop at 1/2 depth within the NDLs of even the old USN tables maintains the phase volumes below limit points;
5. the LANL 450/20 profiles also surface below the phase volume limit point, no surprise because profiles were designed to meet that constraint;
6. supersaturation profiles MN90 and NEDU 2 also do not control bubble growth in the deeper zones, but the separated phase volume is below model limit points, with pressure in the shallow zone sufficient to constrain bubble growth and maintain adequate dissolution, but time consuming because bubbles are now larger in the shallow zone.

Much the same can be said of supersaturation versus bubble staging strategies in general.

APPENDIX B

RGBM User Overview and Statistics

Training Agencies, particularly NAUI, ANDI, Finnish Diving Federation (FDF), and Irish Diving Federation (IDF) employ both RGBM Tables and dive planning software in their formal course structures, as part of standards and procedures and extended range training. Commercial operations in the Gulf and elsewhere are eyeing RGBM trimix and heliox tables for deep work, and air tables for shallow activities (less than 140 *fsw*). Many RGBM table dives are stored in the LANL Data Bank, coming from diverse sectors. Tables enjoy a safe and utilitarian record across mixed gas diving. Though not recorded directly in the LANL Data Bank, save a few select technical, mixed gas, decompression profiles, meter vendor reported usage statistics on both recreational and technical RGBM decompression meters also underscore safe application and utility. On the recreational side, Suunto, Mares, Dacor, UTC, Plexus, and Zeagle market RGBM meters. On the technical side, HydroSpace, Steam Machines, and Atomic Aquatics market, or will shortly market, RGBM meters. Commercial RGBM software includes ABYSS, GAP, and EXPLORER RGBM Simulator. Upgrades to technical RGBM are in the works for these manufacturers, and other new recreational and technical meters are in development and planning stages. Here at LANL, we use special decompression modules in conjunction with dive planning software incorporating the LANL bubble model. Commercial implementations of the RGBM tend to be more conservative than the LANL inhouse version. Consider some usage statistics furnished by vendors and Training Agencies. The compilation only includes respondents at writing. Across this spectrum of recreational and technical diving, less than 98 cases of DCS have been reported or noted. Certainly, and based on observations of many in the diving community, many cases of DCS go unreported.

RGBM Decompression Meters

In recreational circles, computers are mainstay diving tools. In technical mixed gas diving, computers are usually backup or bailout for tables or dive planning software schedules:

Suunto – recreational and light deco air and nitrox meters, 9, 200, 000 dives;

Mares – recreational and light deco air and nitrox meters, 2, 200, 000 dives;

Dacor – recreational and light deco air and nitrox meters, 450, 000 dives;

HydroSpace – technical mixed gas OC and RB deco meters, 45, 000 dives;

UTC – recreational and light deco air and nitrox meters, not available at writing;

Plexus – recreational and light deco air and nitrox meters, not available at writing;

Steam Machines – constant pp_{O_2} deco meters, not available at writing;

Zeagle – recreational and light deco air and nitrox meters, not available at writing;

Atomic Aquatics – technical mixed gas OC and RB deco meters, under development.

RGBM Software

ABYSS, GAP, ANDI GAP, NAUI GAP, Ocean Concepts, and HydroSpace RGBM Simulator are dive planning software packages used mainly by technical divers across commercial, research, and exploration sectors. Combined estimates of packages marketed by the six is presently 12, 000+. Only a few scattered reports of DCS have been reported or noted among diver users of these software packages. These modules run on desktop and laptop computers, which in the latter case, are often taken to, and used at, the dive site.

RGBM Training Agencies

NAUI, ANDI, FDF, and IDF formally incorporate RGBM schedules, software, and tables into their training regimens. Rough statistics suggest:

NAUI – recreational and light deco air and nitrox tables, 514, 000 dives;

NAUI – technical mixed gas decompression tables, 26, 000 dives;

NAUI – NAUI GAP dive planner, 5, 700 dives;

ANDI – ANDI GAP dive planner, 5, 000 dives;

FDF – recreational and light deco air and nitrox tables, not available at writing;

IDF – recreational and light deco air and nitrox tables, under analysis.

RGBM Tables

RGBM Tables for air, nitrox, helitrox, trimix, and heliox are used by Training Agencies, technical, commercial, scientific, and exploration divers, and span OC to RB mixed gas diving. These tend toward the conservative side of the LANL model. In addition to the 2, 000, 000+ Training Agency dives on RGBM Tables, another 20, 000 -30, 000 dives might be expected from trained divers. The use of GAP software by NAUI and ANDI is a recent development over the past 3 -4 years.

APPENDIX C

Software and Parallel Implementation

A rundown of the LANL (data correlated) software configuration of the RGBM used in analyses is tabulated. The package is under constant refinement and updating, and can be used on open circuit (OC) or rebreather (RB) systems. It has been a mainstay in dive planning and operations here at LANL. Parameters in the model and software have been calibrated against profile outcomes in the LANL Data Bank. The same module is used to generate bubble and dissolved gas risk functions employed in likelihood analysis of data. The package has been licensed commercially, put into decompression meters, and tailored for individual needs:

1. Module: integrated bubble excitation, dissolved gas and bubble gas transfer, material equations of state for surfactants, Boyle expansion and contraction, and staging routines, with waypoints prior to ascent, for nitrox, heliox, and trimix.
2. Source Code: 1640 Lines.
3. Language/Compiler: FORTRAN 77/90, BASIC.
4. CRAY YMP Running Time: 1 *sec* for deep trimix profile with 5 gas switches on way up.
5. Input: altitude, bottom mixture/pp_{O_2}, ascent/descent rate, switch levels and gas mixtures/pp_{O_2}s, predive breathing gas, safety knobs, previous dive history.
6. Output: controlling tissue compartments, stop depth and times, supersaturation gradient, permissble supersaturation, effective bubble and gas parameters, critical phase volume, dive profile.

Commercial versions are marketed by GAP, ABYSS, and HydroSpace Engineering. Meter implementations are marketed by Suunto, Mares, HydroSpace, Dacor, Plexus, Zeagle, Steam Machines, UTC, and others in the works.

The enormous computing power and lightning speed of the LANL Blue Mountain MPP (massively parallel processor) permits fast and compute intensive numerical experiments with data. So as a variance reduction technique across the full canonical data set, using a random number generator for profiles across 2, 000 parallel SMP (Origin 2000) processors at LANL, we construct 2, 000 subsets, with K = 750 across $p \leq 0.0069$, for separate likelihood analysis, weighting each processor κ, λ, γ, and μ by the number of sample hits divided by the number of population hits. This cuts run and analysis time, plus numerical roundoff errors implicit to likelihood analysis for small *r*, and large *K*. The sorting continues through all possible profile combinations, χ, roughly,

$$\chi \propto \frac{2879!}{750!2109!}$$

which is a very large set of calculational samples for any computer, save massively parallel, very fast, large core machines available at select locations in the world. Processors with zero DCS hits in the sample contribute nothing to the total tally. Such a weighting technique has tremendous advantages in Monte Carlo applications, providing fast and reliable estimates of statistical quantities over condensed event space. At LANL, major gains are seen in particle transport, hydrodynamic, and plasma applications of Monte Carlo techniques. The method is similar to roulette, biasing, importance sampling, splitting, and other variance reduction techniques utilized in transport phenomenology. Recall that the Blue Mountain MPP boasts overall processor speeds in the teraflop range (10^{12} *binary operations/sec*). The massively compute intensive program above takes some 30 - 40 *sec*.

APPENDIX D

Field Data

Models need validation and field testing. Often, strict hyperbaric chamber tests are not possible, economically nor otherwise, and models employ a number of benchmarks and regimens to underscore viability. The following are some supporting validation of the RGBM phase model and (released) nitrox, heliox, and trimix diving tables, software, and meters. Profiles are recorded in the LANL Data Bank, and are representative of entries in terms of dive counts and technical diving applications:

1. Counterterror and Countermeasures Team (C & C) RB and OC exercises have used the RGBM (iterative deep stop version) for a number of years, logging some 2324 dives on mixed gases (trimix, heliox, nitrox) with 0.4% incidence of DCS – 85% were deco dives, and 55% were repets with at least 2 hr SIs, with most in the forward direction (deepest dives first). Some 14 cases of DCS were logged by the Team, mainly in the deep reverse profile category on nitrox and trimix, plus RB hits on heliox;

2. NAUI Technical Diving has been diving the deep stop version for the past 9 yrs, some estimated 22, 000 dives, on mixed gases down to 300 *fsw*, with2 reported cases ofDCS, bothontrimix. Some 15 divers, late 1999, in France used the RGBM to make 2 mixed gas dives a day, without mishap, in cold water and rough seas. Same thing in the warm waters of Roatan in 2000, 2001, 2002, 2003, 2004, 2005, and 2006;

3. within above 2324 dives, a series of 20 dives to 450 *fsw* for 20 *min* on RBs, setpoint 1.4 *atm*, was performed without malaise in a population of male and female divers, ages 25 -68 *yrs*;

4. NAUI Worldwide released a set of RGBM Tables for air, EAN32, and EAN36 recreational diving, from sea level to 10, 000 ft, a few years ago. Minimum SIs of 1 hour are supported for repetitive diving in all Tables, and safety stops for 2 *min* in the 15 *fsw* zone, plus 1 min deep stops at half bottom depth, are required always. Tables were tested by NAUI Insructor Trainers, Instructors, and Divemasters over a 2 year period without mishap, and continue so today as mainstay teaching Tables in NAUI basic air and nitrox courses;

5. modified RGBM recreational algorithms (Haldane imbedded with bubble reduction factors limiting reverse profile, repetitive, and multiday diving), as coded in Suunto, Mares, Dacor, UTC, Zeagle, Steam Machines, GAP, ABYSS, HydroSpace, Plexus decometers, maintain an already low DCS incidence rate of approximately 1/50, 000 or less. More RGBM decompression meters, including mixed gases, are in the works;

6. a cadre of divers and instructors in mountainous New Mexico, Utah, and Colorado have been diving the modified RGBM at altitude, an estimated 1, 200 dives, without peril. Again, not surprising since the altitude RGBM is slightly more conservative than the usual Cross correction used routinely up to about 8, 000 ft elevation, and with estimated DCS incidence less than 1/10, 000;

7. within decometer implementations of the RGBM, only a few scattered DCS hits have been reported in nonstop and multidiving categories, beyond 1, 300, 000 dives or more, up to now, according to statistics furnished the author (BRW) by meter vendors;

8. extreme hyperbaric chamber tests for mixed gas RGBM protocols are in the works, and less stressful exposures will be addressed also – extreme here means 300 *fsw* and beyond;

9. as seen, probabilistic decompression analysis of selected recreational air RGBM profiles, calibrated against similar calculations of the same profiles by Duke, help validate the RGBM on computational bases, suggesting the RGBM has no more theoretical risk than other bubble or dissolved gas models (Weathersby methodology at USN and Duke);

10. all divers and Instructors using RGBM decometers, tables, or Internet software have been asked to report individual profiles to DAN Project Dive Exploration (Vann, Denoble at Duke), plus to the LANL Data Bank (Wienke, O'Leary at LANL and NAUI);

11. GAP, HydroSpace RGBM Simulator, and ABYSS are NET software packages that offer the modified RGBM (folded Buhlmann ZHL) and, especially, the full up, deep stop version for any gas mixture, have a fairly large contingent of tech divers already using the RGBM and have not received any reports of DCS to date. The EXPLORER RGBM Simulator is furnished to meter owners of the HydroSpace EXPLORER;

12. extreme WKPP profiles in the 300 *fsw* range on trimix were used to calibrate the RGBM. WKPP profiles are the most impressive application of RGBM staging, with

as much as 6 hours less decompression time for WKPP helium based diving on RGBM schedules versus Haldane schedules, with estimated 200 dives;

13. Ellyat, a TDI Instructor, dived the Baden in the North Sea to 540 *fsw* on RGBM Tables on two different occasions, and 3 hours were shaved off conventional hang time by RGBM application. Unfortunately, with diver error and mismatched gas switching strategies from helium to nitrogen, dives to 840 *fsw* resulted in vestibular DCS;
14. NAUI Worldwide released sets of deep stop RGBM nitrox, heliox, and trimix technical and recreational Tables that have been tested by NAUI Technical Diving Operations over the past 9 years, with success and no reported cases of DCS, for open circuit regulators and rebreathers;
15. Doppler and imaging tests in the laboratory, and analyses by Marroni, Bennett, Brubakk and Wienke, and Neuman all suggest reduction in free phase counts with deep stop staging;
16. deep air RGBM Tables with surface oxygen decompression are employed by American oil patch diving companies;
17. Scorese, a NAUI instructor, and his students made a total of 234 dives on the Andrea Doria using rebreathers and RGBM (constant pp_{O_2}) RB Tables, and various nitrogen and trimix diluents. Dive abortions off rebreathers employed ranged RGBM (open circuit) Tables as bailouts, and witnessed no mishaps;
18. Freauf, a Navy SEAL in Hawaii, logged 20 trimix decompression dives beyond 250 *fsw* on consecutive days using RGBM Tables (pure oxygen switch at 20 *fsw*);
19. Melton, owner of HydroSpace Engineering and developer of the RGBM EXPLORER (OC plus RB) dive computer reports 100s of dives in the 400 *fsw* range on the RGBM EXPLORER;
20. GAP, Gas Absorption Program, an RGBM software product out of the Netherlands, supports brisk and sustained use of the RGBM within the tec and rec diving community;
21. heliox RGBM Tables are being used by a commercial diving operation in Argentina;
22. the RGBM EXPLORER is also employed in scattered commercial diving operations;
23. Raine, a wreck diver in California, reports 100s of RGBM dives in the 250 *fsw* range with low Doppler counts;
24. ANDI, a training agency, has adopted a custom version of GAP for diver training on mixed gases, OC and RBs;
25. the Israeli Navy utilizes the ANDI GAP RGBM dive planner;
26. NAUI similarly employs a custom version of GAP for dive planning, with nominal GAP parameter settings recovering released and published NAUI RGBM Tables;
27. O'Leary, Director NAUI Technical Operations, has made over 170 dives on OC and RB systems using RGBM Tables and the HydroSpace EXPLORER to depths beyond 250 *fsw*, with anywhere from 6 -9 other divers during NAUI Technical Instructor Training Courses; O'Leary, Sharp, Scorese, Bell, Hunley, and 6 other NAUI Instructors used RGBM OC and RB Tables to dive the USS Perry in Anguar in very strong currents, down to 260 *fsw*, logging 2 repetitive deco dives a day for a week or so;
28. the Finnish Diving Federation (FDF) has adopted RGBM Tables for recreational air and nitrox diver training, as well as light decompression exposures down to 130 *fsw*;
29. the Irish Diving Federation is inspecting RGBM recreational and light decompression air and nitrox tables.

While the foregoing list of field tests and profiles are not controlled scientific experiments, the shear number of diving events and diversity of exposure spectrum ought not be discounted. Collective information has been dubbed a *living laboratory* by segments of the technical, scientific, and operational diving community. Concurrently, it is noted that DCS spikes among table, meter, and software users have not been seen nor reported by divers, meter vendors, training agencies, and commercial operations using RGBM scheduling, nor any other deep stop algorithm for that matter.

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